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不确定终止时间的多阶段最优投资组合①

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摘要: 研究了当终止时间不确定时的多阶段最优投资组合问题. 假定终止时间是个服从某分布的随机变量, 将不确定终止时间的问题转化为确定时间的问题, 应用动态规划求解模型, 得到最优投资策略以及有效边界的解析形式. 实例证明所得的结论是对确定终止时间情形的推广, 最优投资策略受终止时间分布的影响.

关键词: 不确定终止时间; 多阶段; 动态规划; 最优投资策略
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0 引 言

Markowitz 创立单阶段 M - V 投资组合理论以来, 投资组合理论的发展非常迅速. 周奕等[1]研究了有资本结构因子作用的投资组合问题; 刘海龙等[2]应用效用方法研究了有交易费的投资组合问题; 李仲飞[3]、Li 与 Ng[4]、Li、Chan 与 Ng[5]、Fama[6]、Hakansson[7]、Elton 与 Gruber[8]研究了多阶段最优投资组合问题. 但是, 文献[3～8]都假定终止时间是个确定常数. 在实际中, 一个投资者事先拟定一个多阶段的投资计划, 但是在投资过程中, 可能因为意外事情导致投资计划未完成而中途终止. 这个中途终止时间不是确定的, 是个随机变量或者更一般是个随机过程. 这类投资问题, 称为随机时域的投资问题. 连续时间的随机时域问题已有许多研究. Merton[9,10]研究了当退出时间是个 Possion 过程的投资消费问题. Merton 发现, 若终止财富为零时, 这个问题与一个无限时域的问题等价. Karatzas 与 Wang[11]研究了退出时间是个价格滤波下的停时的最优投资消费问题. 在该情形下, 该问题可以转化为一个最优停时问题. 从文献[9～11]的结论来看, 随机终止时间会影响连续时间的最优投资策略的选择. 也就是说确定时域下得到的结论在随机时域下不一定成立.

本文在假定投资者的终止时间服从某个离散分布的情况下, 研究了一个不确定终止时间的离散最优投资问题. 以资产增长倍数的期望最大方差最小为目标建立模型. 将随机时域问题转化为确定时域问题, 应用动态规划方法求解模型, 得到最优投资策略以及有效边界.

1 模 型

设投资者从 0 时刻进入市场投资, 其初始财富为 W_0, 计划进行 T 个阶段的投资. 市场上有 $n+1$ 种证券, 其中 1 种无风险证券, n 种风险证券, 其在 t 阶段的收益分别为 $r_t^0, r_t^1, \cdots, r_t^n$. 记 $r_t = (r_t^1, r_t^2, \cdots, r_t^n)'$, $R_t^i = r_t^i - r_t^0$, $i = 1, 2, \cdots, n$, $R_t = (R_t^1, R_t^2, \cdots, R_t^n)'$, 其中上标“ ′”表示转置. 设 x_t^i 为投资者在 t 阶段投资到第 t 个风险证券上的资金比例, 则在 t 阶段投资到无风险证券上的资金比例为 $1 - \sum_{i=1}^{n} x_t^i$, 记 $x_t = (x_t^1, x_t^2, \cdots, x_t^n)'$, 称 x_t 为第 t 阶段的一个投资组合, 从开始投资到退出, 各个阶段投资组合的集合称为一个投资策略, 记为 u, 即 $u = \{x_1, x_2, \cdots, x_T\}$. 设投资者

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实际只进行了 τ 个阶段的投资. 在这里假定 τ 是个概率分布为 $p_0 = 0, p_i = p(\tau = i), (i = 1,2,\cdots)$, 分布函数为 $F(t)$ 的随机变量, 则 $F(t) = \sum_{i=0}^{t} p_i$.

令 v_t 为投资者第 t 个阶段投资结束时的财富相对于初始财富的增长倍数,则

$$v_0 = 1, v_t = v_{t-1}(r_t^0 + R_t' x_t), t = 1,2,\cdots,T$$

投资者的目的就是在随机时域内,找到一个投资策略使得最终财富增长倍数的期望最大,风险最小. 于是建立模型为

$$\begin{cases} \max_u E(v_{T\wedge\tau}) - \omega Var(V_{T\wedge\tau}) \\ \text{s.t.} \quad v_t = v_{t-1}(r_t^0 + R_t' x_t) \end{cases} \quad (1)$$

其中:参数 $\omega > 0$, 记 $\Pi_\omega = \{u \mid u$ 是(1)的最优投资策略$\}$.

定义 一个投资策略称为是有效的,如果不存在其它投资策略 u,使得

$$E(v_t^u) \geq E(v_t^{u^*}), Var(v_t^u) \leq Var(v_t^{u^*})$$

而且至少有一个不等式严格成立,其中,v_t^u 表示在投资策略 u 下,第 t 个阶段结束时的财富相对于初始财富的倍数. 所有有效投资组合对应的点 $(E(v_{T\wedge\tau}), Var(v_{T\wedge\tau}))$ 的集合称为有效前沿. 引进近似问题

$$\begin{cases} \max_u E(\lambda v_{T\wedge\tau} - \omega v_{T\wedge\tau}^2) \\ \text{s.t.} \quad v_t = v_{t-1}(r_t^0 + R_t' x_t) \end{cases} \quad (2)$$

其中,参数 $\omega > 0$; $\lambda \in (-\infty, \infty)$. 记 $\Pi_{(\lambda,\omega)} = \{u \mid u$ 是(2)的最优投资策略$\}$,则由文献[1]定理5.3.1、定理5.3.2, Π_ω 与 $\Pi_{(\lambda,\omega)}$ 有如下关系

引理1 若 $u^* \in \Pi_\omega$,则 $u^* \in \Pi_{(\lambda,\omega)}$. 其中, $\lambda^* = 1 + 2\omega E(v_{T\wedge\tau}^{u^*})$.

2 最优投资策略

将式(2)转化为一个确定时域的问题,然后用动态规划求解.

由文献[10]可得

$$E(\lambda v_{T\wedge\tau} - \omega v_{T\wedge\tau}^2) = E\Big[\sum_{t=1}^{\infty}(\lambda v_{T\wedge t} - \omega v_{T\wedge t}^2) p_t\Big] = E\Big[\sum_{t=1}^{T-1}(\lambda v_t - \omega v_t^2) p_t + (\lambda v_T - \omega v_T^2)(1 - F(T-1))\Big]$$

于是式(2)等价于下面确定时域的问题

$$\begin{cases} \max_u E\Big[\sum_{t=1}^{T-1}(\lambda v_t - \omega v_t^2) p_t + (\lambda v_T - \omega v_T^2)(1 - F(T-1))\Big] \\ \text{s.t.} \quad v_t = v_{t-1}(r_t^0 + R_t' x_t) \end{cases} \quad (3)$$

令

$$f_{T,T}(v_T) = (\lambda v_T - \omega v_T^2)(1 - F(T-1))$$

$$f_{t,T}(v_t) = \max_{x_{t+1}} E\Big[\sum_{s=t}^{T-1}(\lambda v_s - \omega v_s^2) p_s + (\lambda v_T - \omega v_T^2)(1 - F(T-1))\Big]$$

$$t = 1,2,\cdots,T-1$$

则

$$\begin{aligned} f_{T-1,T}(v_{T-1}) &= \max_{x_T} E\Big[(\lambda v_{T-1} - \omega v_{T-1}^2) p_{T-1} + f_{T,T}(v_T)\Big] = \\ &\max_{x_T} E\Big[(\lambda v_{T-1} - \omega v_{T-1}^2) p_{T-1} + f_{T,T}(v_{T-1}(v_T^0 + R_T' x_T))\Big] = \\ &(\lambda v_{T-1} - \omega v_{T-1}^2) p_{T-1} + \\ &\lambda r_T^0 (1 - F(T-1)) v_{T-1} - \\ &\omega(1 - F(T-1))(r_T^0)^2 v_{T-1}^2 + \\ &\max_{x_T}\{[\lambda(1 - F(T-1)) v_{T-1} - \\ &2\omega r_T^0 (1 - F(T-1)) v_{T-1}^2] \cdot \\ &(ER_T)' x_T - \omega(1 - F(T-1)) v_{T-1}^2 x_T' \cdot \\ &E(R_T R_T') x_T\} \end{aligned} \quad (4)$$

在式(4)的大括号内对 x_T 求梯度,并令其等于零,得

$$[\lambda(1 - F(T-1)) v_{T-1} - 2\omega r_T^0 (1 - F(T-1)) v_{T-1}^2] ER_T - 2\omega(1 - F(T-1)) v_{T-1}^2 E(R_T R_T') x_T = 0$$

解之,得

$$x_T = [E(R_T R_T')]^{-1} ER_T\Big(\frac{\lambda}{2\omega v_{T-1}} - r_T^0\Big) \quad (5)$$

再将式(5)代入 $f_{T-1,T}(v_{T-1})$,得

$$f_{T-1,T}(v_{T-1}) = -A_{T-1} v_{T-1}^2 + B_{T-1} v_{T-1} + C_{T-1}$$

其中

$$A_{T-1} = \omega[p_{T-1} + (1 - F(T-1))(1 - \gamma_T) r_T^{0\,2}]$$

$$B_{T-1} = \lambda[p_{T-1} + (1 - F(T-1))(1 - \gamma_T) r_T^0]$$

$$C_{T-1} = \frac{\lambda^2}{4\omega}(1 - F(T-1))\gamma_T$$

$$\gamma_T = (ER_T)'[E(R_T R_T')]^{-1}(ER_T)$$

$$\begin{aligned} f_{T-2,T}(v_{T-2}) &= \max_{x_{T-1}} E[(\lambda v_{T-2} - \omega v_{T-2}^2) p_{T-2} + f_{T-1,T}(v_{T-1})] = \max_{x_{T-1}} E[(\lambda v_{T-2} - \omega v_{T-2}^2) p_{T-2} - \\ & A_{T-1} v_{T-2}^2 (r^0_{T-1} + R_{T-1}' x_{T-1})^2 + B_{T-1} v_{T-2}(r^0_{T-1} + R_{T-1}' x_{T-1}) + C_{T-1}] = \\ & (\lambda v_{T-2} - \omega v_{T-2}^2) p_{T-2} - A_{T-1} r^{0\,2}_{T-1} v_{T-2}^2 + B_{T-1} r^0_{T-1} v_{T-2} + C_{T-1} + \max_{x_{T-1}}\{[-2A_{T-1} r^0_{T-1} v_{T-2}^2 + \\ & B_{T-1} v_{T-2}](ER_{T-1})' x_{T-1} - A_{T-1} v_{T-2}^2 x_{T-1}' E(R_{T-1} R_{T-1}') x_{T-1}\} = 0 \end{aligned} \tag{6}$$

在式(6)的大括号内对 x_{T-1} 求梯度,并令其等于零,得

$$(-2A_{T-1} r^0_{T-1} v_{T-2}^2 + B_{T-1} v_{T-2}) ER_{T-1} - 2A_{T-1} v_{T-2}^2 E(R_{T-1} R_{T-1}') x_{T-1} = 0$$

解之,得

$$x_{T-1} = [E(R_{T-1} R_{T-1}')]^{-1} ER_{T-1} \cdot \left(\frac{B_{T-1}}{2A_{T-1} v_{T-2}} - r^0_{T-1}\right) \tag{7}$$

再将式(7)代回 $f_{T-2,T}(v_{T-2})$,得

$$f_{T-2,T}(v_{T-2}) = -A_{T-2} v_{T-2}^2 + B_{T-2} v_{T-2} + C_{T-2} + C_{T-1}$$

其中

$$A_{T-2} = \omega p_{T-2} + A_{T-1}(1 - \gamma_{T-1})(r^0_{T-1})^2$$

$$B_{T-2} = \lambda p_{T-2} + B_{T-1}(1 - \gamma_{T-1}) r^0_{T-1}$$

$$C_{T-2} = \frac{B_{T-1}^2}{4A_{T-1}} \gamma_{T-1}$$

$$\gamma_{T-1} = (ER_{T-1})'[E(R_{T-1} R_{T-1}')]^{-1}(ER_{T-1})$$

由此,对 $t \in [1,2,\cdots,T]$,令

$$x_t = [E(R_t R_t')]^{-1} ER_t \left(\frac{B_t}{2A_t v_{t-1}} - r_t^0\right) \tag{8}$$

$$f_{t-1,T}(v_{t-1}) = -A_{t-1} v_{t-1}^2 + B_{t-1} v_{t-1} + \sum_{s=t-1}^{T-1} C_s \tag{9}$$

其中

$$A_{t-1} = \omega p_{t-1} + A_t(1 - \gamma_t)(r_t^0)^2 \tag{10}$$

$$B_{t-1} = \lambda p_{t-1} + B_t(1 - \gamma_t) r_t^0 \tag{11}$$

$$C_{t-1} = \frac{B_t^2}{4A_t} \gamma_t$$

$$\gamma_t = (ER_t)'[E(R_t R_t')]^{-1}(ER_t)$$

其中

$$\begin{aligned} A_T &= \omega(1 - F(T-1)) \\ B_T &= \lambda(1 - F(T-1)) \end{aligned} \tag{12}$$

则

$$\begin{aligned} f_{t-2,T}(v_{t-2}) &= \max_{x_{t-1}} E[(\lambda v_{t-2} - \omega v_{t-2}^2) p_{t-2} + f_{t-1,T}(v_{t-1})] = \max_{x_{t-1}} E[(\lambda v_{t-2} - \omega v_{t-2}^2) p_{t-2} - \\ & A_{t-1} v_{t-2}^2 (r^0_{t-1} + R_{t-1}' x_{t-1})^2 + B_{t-1} v_{t-2}(r^0_{t-1} + R_{t-1}' x_{t-1}) + \sum_{s=t-1}^{T-1} C_s] = \\ & (\lambda v_{t-2} - \omega v_{t-2}^2) p_{t-2} - A_{t-1}(r^0_{t-1})^2 v_{t-2}^2 + B_{t-1} r^0_{t-1} v_{t-2} + \sum_{s=t-1}^{T-1} C_s + \\ & \max_{x_{t-1}}\{[-2A_{t-1} r^0_{t-1} v_{t-2}^2 + B_{t-1} v_{t-2}](ER_{t-1})' x_{t-1} - A_{t-1} v_{t-2}^2 x_{t-1}' E(R_{t-1} R_{t-1}') x_{t-1}\} = 0 \end{aligned} \tag{13}$$

在式(13)的大括号内对 x_{t-1} 求梯度,并令其等于零,得

$$(-2A_{t-1} r^0_{t-1} v_{t-2}^2 + B_{t-1} v_{t-2}) ER_{t-1} - 2A_{t-1} v_{t-2}^2 E(R_{t-1} R_{t-1}') x_{t-1} = 0$$

解之,得

$$x_{t-1} = [E(R_{t-1} R_{t-1}')]^{-1} \cdot ER_{t-1}\left(\frac{B_{t-1}}{2A_{t-1} v_{t-2}} - r^0_{t-1}\right) \tag{14}$$

再将式(14)代回 $f_{t-2,T}(v_{t-2})$,得

$$\begin{aligned} f_{t-2,T}(v_{t-2}) &= (\lambda v_{t-2} - \omega v_{t-2}^2) p_{t-2} - A_{t-1}(r^0_{t-1})^2 v_{t-2}^2 + B_{t-1} r^0_{t-1} v_{t-2} + \sum_{s=t-1}^{T-1} C_s + [-2A_{t-1} r^0_{t-1} v_{t-2}^2 + \\ & B_{t-1} v_{t-2}] \gamma_{t-1}\left(\frac{B_{t-1}}{2A_{t-1} v_{t-2}} - r^0_{t-1}\right) - A_{t-1} v_{t-2}^2 \gamma_{t-1}\left(\frac{B_{t-1}}{2A_{t-1} v_{t-2}} - r^0_{t-1}\right)^2 = \\ & -[\omega p_{t-2} + A_{t-1}(1 - \gamma_{t-1})(r^0_{t-1})^2] v_{t-2}^2 + [\lambda p_{t-2} + B_{t-1}(1 - \gamma_{t-1}) r^0_{t-1}] v_{t-2} + \\ & \frac{B_{t-1}^2}{4A_{t-1}} \gamma_{t-1} + \sum_{s=t-1}^{T-1} C_s = -A_{t-2} v_{t-2}^2 + B_{t-2} v_{t-2} + \sum_{s=t-2}^{T-1} C_s \end{aligned}$$

由数学归纳原理,对任何 $t \in \{1,2,\cdots,T\}$ 都

有式(8)、(9)成立.

对式(10)、(11)进行反复迭代,可得

$$A_t = \omega I_t, B_t = \lambda J_t, t = 0,1,2,\cdots,T \quad (15)$$

其中

$$I_t = p_t + \sum_{i=t}^{T-2} p_{i+1}\Big[\prod_{j=t}^{i}(r_{j+1}^0)^2(1 - y_{j+1})\Big] + (1 - F(T-1))\prod_{j=t}^{T-1}(r_{j+1}^0)^2(1 - y_{j+1})$$

$$t = 0,1,2\cdots,T-1$$

$$I_T = 1 - F(T-1) \quad (16)$$

$$J_t = p_t + \sum_{i=t}^{T-2} p_{i+1}\Big[\prod_{j=t}^{i} r_{j+1}^0(1 - y_{j+1})\Big] + (1 - F(T-1))\prod_{j=t}^{T-1} r_{j+1}^0(1 - y_{j+1})$$

$$t = 0,1,2,\cdots,T-1$$

$$J_T = 1 - F(T-1) \quad (17)$$

将式(8)代入 $v_t = v_{t-1}(r_t^0 + R_t' x_t)$, $t = 1,2,\cdots,T$,得

$$v_t = v_{t-1}\Big[r_t^0 + R_t'(E(R_tR_t'))^{-1} \cdot ER_t\Big(\frac{B_t}{2A_tv_{t-1}} - r_t^0\Big)\Big]$$

$$t = 1,2,\cdots,T \quad (18)$$

两边取期望,得

$$Ev_t = \frac{B_t}{2A_t}y + r_t^0(1 - y)Ev_{t-1}$$

$$t = 1,2,\cdots,T \quad (19)$$

对式(19)反复迭代,可得

$$\begin{cases} Ev_1 = \dfrac{B_1}{2A_1}y + r_1^0(1 - y) \\ Ev_t = \dfrac{B_t}{2A_t}y = \displaystyle\sum_{i=1}^{t-1}\frac{B_i}{2A_i}y \cdot \Big[\prod_{j=i}^{t-1} r_{j+1}^0(1 - y_{j+1})\Big] + \prod_{i=1}^{t} r_i^0(1 - y) \\ t = 1,2,\cdots,T \end{cases} \quad (20)$$

由文献[10]以及式(20),得

$$Ev_{T\wedge\tau} = E\Big[\sum_{s=1}^{T-1} v_sp_s + v_T(1 - F(T-1))\Big] = \sum_{s=1}^{T-1} p_sEv_s + (1 - F(T-1))Ev_T = \frac{\lambda}{2\omega}G_T + H_T$$

其中

$$G_T = \sum_{s=1}^{T-1}\frac{J_s}{I_s}yp_s + \sum_{s=2}^{T-1}\Big[\sum_{i=1}^{s-1}\frac{J_i}{I_i}y\sum_{j=i}^{s-1} r_{j+1}^0(1 - y_{j+1})\Big]p_s + \Big[y + \sum_{i=1}^{T-1}\frac{J_i}{I_i}y\prod_{j=i}^{T-1} r_{j+1}^0(1 - y_{j+1})\Big] \cdot (1 - F(T-1)) \quad (21)$$

$$H_T = \sum_{s=1}^{T-1}\Big[\prod_{i=1}^{s} r_i^0(1 - y)\Big]p_s + \prod_{i=1}^{T} r_i^0(1 - y)(1 - F(T-1)) \quad (22)$$

综上所述,下面结论成立.

引理 2 对给定的参数 $\omega > 0$, $\lambda \in$ [illegible],近似问题(2)的最优投资策略为

$$u^* = (x_1^*, x_2^*, \cdots, x_{T\wedge\tau}^*) \quad (23)$$

其中

$$x_t^* = [E(R_tR_t')]^{-1}ER \cdot \Big(\frac{\lambda J_t}{2\omega I_tv_{t-1}} - r_t^0\Big)$$

$$t = 1,2,\cdots,T\wedge\tau$$

其中,I_t、J_t, $t = 1,2,\cdots,T\wedge\tau$,由式(16)、(17)给出.

最终财富增长倍数的期望为

$$Ev_{T\wedge\tau} = \frac{\lambda}{2\omega}G_T + H_T \quad (24)$$

其中,G_T、H_T 由式(21)、(22)给出.

解关于 λ^* 的方程

$$\lambda^* = 1 + 2\omega Ev_{T\wedge\tau}\Big|_{\lambda=\lambda^*} = 1 + 2\omega\Big(\frac{\lambda^*}{2\omega}G_T + H_T\Big)$$

得

$$\lambda^* = \frac{1 + 2\omega H_T}{1 - G_T}$$

将 λ^* 代入式(23)、(24),根据引理 1,得到本文的主要定理.

定理 1 对给定的参数 $\omega > 0$,原问题(1)的最优投资策略为

$$u^* = (x_1^*, x_2^*, \cdots, x_{T\wedge\tau}^*)$$

其中

$$x_t^* = [E(R_tR_t')]^{-1}ER_t \cdot \Big[\frac{(1 + 2\omega H_T)J_t}{2\omega(1 - G_T)I_tv_{t-1}} - r_t^0\Big],$$

$$t = 1,2,\cdots,T\wedge\tau \quad (25)$$

其中, I_t、J_t, $t=1,2,\cdots,T\wedge\tau$, 由式(16)、(17)给出.

最终财富增长倍数的期望为

$$Ev_{T\wedge\tau}=\frac{1+2\omega H_T}{2\omega(1-G_T)}G_T+H_T \tag{26}$$

其中, G_T、H_T 由式(21)、(22)给出.

注: 当投资者的退出时间为确定时间 T 时,可以令 τ 的概率分布为 $p(\tau=i)=0, i=1,2,\cdots,T-1, p(\tau=T)=1$. 这时

$$I_t=\prod_{j=t}^{T-1}(r_{j+1}^0)^2(1-y_{j+1}), \quad J_t=\prod_{j=t}^{T-1}r_{j+1}^0(1-y_{j+1})$$

那么

$$\frac{J_t}{I_t}=\frac{1}{\prod_{j=t}^{T-1}r_{j+1}^0}=\frac{1}{\sum_{j=t+1}^{T}r_j^0} \tag{27}$$

$$\begin{aligned}G_T&=y_T+\sum_{i=1}^{T-1}\frac{J_i}{I_i}y_i\sum_{j=i}^{T-1}r_{j+1}^0(1-y_{j+1})=\\&y_T+\sum_{i=1}^{T-1}\frac{1}{\prod_{j=i}^{T-1}r_{j+1}^0}y_i\sum_{j=i}^{T-1}r_{j+1}^0\prod_{j=i}^{T-1}(1-y_{j+1})=\\&y_T+\sum_{i=1}^{T-1}y_i\prod_{j=i}^{T-1}(1-y_{j+1})=\\&1-\prod_{j=1}^{T}(1-y_j)\end{aligned}$$

$$H_T=\prod_{j=1}^{T}r_j^0(1-y_j)$$

从而

$$\frac{H_T}{1-G_T}=\prod_{j=1}^{T}r_j^0 \tag{28}$$

将式(27)、(28)代入(25),得

$$x_t^*=\frac{1}{v_{t-1}}\left[\prod_{j=1}^{t}r_j^0+\frac{1}{2\omega\left[\prod_{j=t+1}^{T}r_j^0\right]\left[\prod_{j=1}^{T}(1-y_j)\right]}\right]\cdot[E(R_tR_t')]^{-1}ER_t-r_t^0[E(R_tR_t')]^{-1}ER_t$$

$$t=1,2,\cdots,T$$

这就是确定终止时间的结论(见文献[1]式(5.5.2)). 因此本文是对确定终止时间情形的一个推广.

如果一个投资者在投资前,拟定好了他退出深刻的期望收益倍数,即 $Ev_{T\wedge\tau}$ 已知,可以通过式(26)解出 2ω,得

$$2\omega=\frac{G_T}{(1-G_T)Ev_{T\wedge\tau}+H_T}$$

代入式(25)即得该投资者的最优投资策略为

$$u^*=(x_1^*,x_2^*,\cdots,x_{T\wedge\tau}^*)$$

其中

$$x_t^*=[E(R_tR_t')]^{-1}ER_t\cdot\left[\frac{Ev_{T\wedge\tau}-H_T}{G_T}\cdot\frac{J_t}{I_t}\cdot\frac{1}{v_{t-1}}-r_t^0\right]$$

$$t=1,2,\cdots,T\wedge\tau \tag{29}$$

由文献[1]式(5.4.23)(式中 B_t 对应本文的 y_t, v_{T+1} 对应本文的 v_T),得

$$\frac{\lambda}{2\omega}=\frac{Ev_T-\prod_{t=1}^{T}r_t^0(1-y_t)}{1-\prod_{t=1}^{T}(1-y_t)}$$

代入文献[1]式(5.4.16),得已知终止期望收益倍数的最优投资策略为

$$x_t^*=[E(R_tR_t')]^{-1}ER_t\cdot\left[\frac{Ev_T-\prod_{t=1}^{T}r_t^0(1-y_t)}{\left[1-\prod_{t=1}^{T}(1-y_t)\right]\prod_{s=t+1}^{T}r_s^0}\cdot\frac{1}{v_{t-1}}-r_t^0\right]$$

$$t=1,2,\cdots,T \tag{30}$$

从式(29)、(30)看出,比较这两种情形各个阶段的投资情况,只需比较 $\frac{Ev_{T\wedge\tau}-H_T}{G_T}\cdot\frac{J_t}{I_t}$ 与 $\frac{Ev_T-\prod_{t=1}^{T}r_t^0(1-y_t)}{\left[1-\prod_{t=1}^{T}(1-y_t)\right]\prod_{s=t+1}^{T}r_s^0}$ 这两项. 为方便起见,考虑一个实例:

一个投资者,初始财富 10 万元,他可选择的投资对象有无风险证券和风险证券. 他计划进行两个阶段的投资,不妨设一个阶段为一年. 这两年里,无风险利率为 $r_1^0=r_2^0=0.1$,风险证券的平均收益率分别为 $Er_1^1=0.2$ 和 $Er_2^1=0.18$, 其风险分别为 $Var(r_1^1)=0.025$ 和 $Var(r_2^1)=0.02$. 则 $ER_1=0.1$, $ER_2=0.08$, $VarR_1=0.025$, $VarR_2=0.02$, 那么

$$y_1=\frac{(ER_1)^2}{E(R_1^2)}=\frac{0.01}{0.025+0.01}=0.285\,71$$

$$y_2=\frac{(ER_2)^2}{E(R_2^2)}=\frac{0.006\,4}{0.02+0.006\,4}=0.242\,42$$

投资者估计在第一年投资结束后,有三成把

据不能进行第二年(阶段) 的投资,但仍有七成把握完成第二年(阶段) 的投资,即 τ 服从两点分布

$$p_1 = p(\tau = 1) = 0.3, p_2 = p(\tau = 2) = 0.7$$

则根据以上公式计算,可得

$$I_1 = p_1 + p_2 (r_2^0)^2 (1 - y) = 0.305\ 00$$

$$J_1 = p_1 = p_2 r_2^0 (1 - y) = 0.350\ 00$$

$$\frac{J_1}{I_1} = 1.147\ 54$$

$$G_2 = \frac{J_1}{I_1} y p_1 + y p_2 + \frac{J_1}{I_1} y r_2^0 (1 - y) p_2 = 0.285\ 44$$

$$H_2 = r_1^0 (1 - y) p_1 + r_1^0 r_2^0 (1 - y)(1 - y) p_2 = 0.025\ 22$$

若投资者拟定自己退出时刻期望收益是初始财富的 1.1 倍,那么

$$\frac{Ev_{2 \wedge \tau} - H_2}{G_2} \cdot \frac{J_1}{I_1} = \frac{1.1 - 0.025\ 22}{0.285\ 44} \times 1.147\ 54 = 4.320\ 88$$

$$\frac{Ev_2 - r_1^0 r_2^0 (1 - y)(1 - y)}{[1 - (1 - y)(1 - y)] r_2^0} = \frac{1.1 - 0.1 \times 0.1 \times 0.714\ 29 \times 0.757\ 58}{(1 - 0.714\ 29 \times 0.757\ 58) \times 0.1} = 23.854\ 10$$

两个数据有显著性差异,这表明,随机终止时间的投资与确定终止时间的不同,也说明,随机终止时间的概率分布会影响投资者对投资策略的选择.

3 有效边界

对式(18) 两边平方,得

$$v_t^2 = \Big[(r_t^0)^2 + 2 r_t^0 R_t' (E(R_t R_t'))^{-1} \cdot ER_t \Big(\frac{B_t}{2A_t v_{t-1}} - r_t^0 \Big) + R_t' (E(R_t R_t'))^{-1} ER_t (ER_t)' \cdot (E(R_t R_t'))^{-1} R_t \Big(\frac{B_t^2}{4A_t^2 v_{t-1}^2} - 2 r_t^0 \frac{B_t}{2A_t v_{t-1}} + (r_t^0)^2 \Big) \Big] v_{t-1}^2$$

两边取期望,得

$$Ev_t^2 = \frac{B_t^2}{4A_t^2} y + (r_t^0)^2 (1 - y) Ev_{t-1}^2 \quad t = 1, 2, \cdots, T \qquad (31)$$

对式(31) 反复迭代,可得

$$\begin{cases} Ev_1^2 = \dfrac{B_1^2}{4A_1^2} y + r_1^{02} (1 - y) \\ Ev_t^2 = \dfrac{B_t^2}{4A_t^2} y + \displaystyle\sum_{i=1}^{t-1} \frac{B_i^2}{4A_i^2} y \Big[\prod_{j=i}^{t-1} (r_{j+1}^0)^2 (1 - y_{+1}) \Big] + \prod_{i=1}^{t} (r_i^0)^2 (1 - y), t = 1, 2, \cdots, T \end{cases} \qquad (32)$$

由文献[10] 以及式(32) ,得

$$Ev_{T \wedge \tau}^2 = E\Big[\sum_{s=1}^{T-1} v_s^2 p_s + v_T^2 (1 - F(T-1)) \Big] = \sum_{s=1}^{T-1} p_s Ev_s^2 + (1 - F(T-1)) Ev_T^2 = \frac{\lambda^2}{4\omega^2} Q_T + U_T$$

其中

$$Q_T = \sum_{s=1}^{T-1} \frac{J_s^2}{I_s^2} y p_s + \sum_{s=2}^{T-1} \Big[\sum_{i=1}^{s-1} \frac{J_i^2}{I_i^2} y \cdot \prod_{j=i}^{s-1} (r_{j+1}^0)^2 (1 - y_{+1}) \Big] p_s + \Big[y + \sum_{i=1}^{T-1} \frac{J_i^2}{I_i^2} y \prod_{j=i}^{T-1} (r_{j+1}^0)^2 (1 - y_{+1}) \Big] \cdot (1 - F(T-1))$$

$$U_T = \sum_{s=1}^{T-1} \Big[\prod_{i=1}^{s} (r_i^0)^2 (1 - y) \Big] p_s + \prod_{i=1}^{T} (r_i^0)^2 (1 - y)(1 - F(T-1))$$

则

$$Var(v_{T \wedge \tau}) = Ev_{T \wedge \tau}^2 - (Ev_{T \wedge \tau})^2 = \Big(\frac{\lambda^2}{4\omega^2} Q_T + U_T \Big) - \Big(\frac{\lambda}{2\omega} G_T + H_T \Big)^2 = \frac{\lambda^2}{4\omega^2} (Q_T - G_T^2) - \frac{\lambda}{\omega} G_T H_T + (U_T - H_T^2)$$

由式(24) ,解得

$$\frac{\lambda}{2\omega} = \frac{Ev_{T \wedge \tau} - H_T}{G_T} \qquad (33)$$

将式(33) 代入 $Var(v_{T \wedge \tau})$ 表达式,得

$$V\alpha(v_{T\wedge\tau}) = \frac{Q_T - G_T^2}{G_T^2}(Ev_{T\wedge\tau} - H_T)^2 - H_T(Ev_{T\wedge\tau} - H_T) + (U_T - H_T^2) \quad (34)$$

由上面推导得到下面定理.

定理 2　不确定退出时间的 M - V 有效边界在均值 - 方差平面上是一条二次曲线,其表达式由式(34) 给出.

4　结束语

本文研究了一个不确定终止时间的多阶段的 M - V 模型,得到了最优投资策略及有效边界,并证实了随机终止时间会影响投资者对投资策略的选择,本文是对确定终止时间情形的一个推广.

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Qualitative simulation for group work behavior

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Abstract : According to QSIM theory , a qualitative simulation method for forecasting and describing group work behavior transition process is set up. Firstly , the qualitative description method for group work behavior and effect relationship between each other is designed. The concepts of decision variable and state variable are given. Secondly , the rule of state variables' transition is defined , based on which all of the possible I transition and P transition are listed , and some transitions are explained by graph. After that , the steps of algorithm are designed. At last , a group work behavior transition process is simulated. It shows that this qualitative simulation method can explain and forecast the transition process of group work behavior.

Key words : group behavior ; qualitative simulation ; QSIM ; decision variable ; state variable

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Multi-period portfolio optimization when exit time is uncertain

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Abstract : The paper studies a multi-period portfolio optimization problem with uncertain exit time. With the assumption that the exit time is a random variable obeying some distribution this problem of uncertain exit time is translated into a determinate horizon one. Then the classical methods can be used to solve this model. By applying the dynamic programming principle we obtain the optimal investment strategy and the analytical expression of efficient frontier. Through an example we also prove that this paper is an extension to determinate horizon case and that the optimal investment strategy is affected by the distribution of exit time.

Key words : uncertain exit time ; multi-period ; dynamic programming ; optimal investment strategy